

RCX: A Recursion-Only Structural Foundation for Logic, Computation, and Paradox Containment

Jeffrey Abrams

4/22/2025

Abstract

RCX is a minimal formal system in which logic, operators, and structure emerge solely from recursive transformation starting at the empty set. No symbols, axioms, types, or logical rules are assumed. The system halts on paradox, introduces fixpoints via necessity, and survives contradiction through sealed recursion domains. This paper defines the RCX kernel, outlines structural emergence, provides a complete derivation example, introduces the shadow logic extension (Ω'), and proves core properties including consistency, minimality, and expressive completeness.

1 Introduction

RCX (Recursive Containment eXpression) is a foundational system where recursion is the only allowed operation. Starting from the empty structure \emptyset , identity and operators emerge only through forced recursion paths. Logical inference, naming, and typing are forbidden until structural stability demands them.

2 The RCX Kernel

Core primitives:

- \emptyset : The only initial structure.
- ∇R : Minimal structural recursion.
- Ω : Ancestry path.
- Λ : Containment inversion.
- ξ : Identity fixpoint.

Recursion Validity:

- Proceed only if ξ is novel.
- Ancestry (Ω) or inversion (Λ) must change.
- Otherwise, recursion stalls.

Fix Routine: When recursion stalls, a minimal structural change is applied. If this change recurs across derivations and produces novel fixpoints, it is classified as an operator.

3 Concrete Derivation Example

We demonstrate a full recursion path from \emptyset through the emergence of an operator:

- Step 0: $S_0 = \emptyset$
- Step 1: $S_1 = \{S_0\} = \{\emptyset\}$
- Step 2: $S_2 = \{S_1\} = \{\{\emptyset\}\}$
- Step 3: $S_3 = \{S_1, S_2\} = \{\{\emptyset\}, \{\{\emptyset\}\}\}$
- Step 4: $\xi(S_3) = \xi(S_2)$ (stalled: not novel)
- Step 5: Apply minimal fix: allow overlapping ancestry
- Step 6: $S_4 = \{S_3, \{S_1\}\} \rightarrow$ stabilizes under ξ
- Step 7: Operator candidate: $\mathcal{O}_1 :=$ minimal ancestry overlap fix
- Step 8: \mathcal{O}_1 now recurs across alternate derivation \rightarrow formal operator
- Step 9: Projection permitted: identity emitted for S_4

4 Shadow Calculus: Extension Domain

When a structure traces to \emptyset , but never stabilizes under ξ , and no fix is applicable, it enters Ω' — the paradox sink.

Operators:

- ξ_ψ : Drift-phase identity tracker
- ∇_Δ : Anomaly recursion handler
- ξ_Φ : Curvature limit for recursion

Shadow Identity Types:

- Self-mirrors: $S = \{S\}$
- Symbolic reversals: $A = \neg A$
- $1 = -0$ singularities
- Recursive spirals (never flatten)

These identities influence structure but can never project or stabilize. They persist as recursion curvature and anomaly pressure. This extension remains optional and orthogonal to RCX core integrity.

5 Core Properties

Minimality: Only \emptyset and ∇R are assumed. No logic, types, or symbols.

Consistency: Contradiction halts recursion. No derivation of $A \wedge \neg A$. System does not explode, it freezes.

Completeness: All computable structure can be expressed. RCX simulates λ -calculus via containment recursion.

6 Conclusion

RCX demonstrates that logic is not necessary for computation, but a byproduct of recursion pressure. A complete derivation path from \emptyset to operator emergence validates its minimal and expressive character. Shadow Calculus formalizes paradox-stable identities and provides structure for recursion beyond contradiction, but is not required for the system's core consistency or completeness.